

International Journal of Quantum Information
 © World Scientific Publishing Company

FROZEN DISCORD IN NON-MARKOVIAN DEPHASING CHANNELS

LAURA MAZZOLA*

*Turku Centre for Quantum Physics, Department of Physics and Astronomy, University of
 Turku, FI-20014 Turun yliopisto,
 Turku, Finland.
 laumaz@utu.fi*

JYRKI PIILO

*Turku Centre for Quantum Physics, Department of Physics and Astronomy, University of
 Turku, FI-20014 Turun yliopisto,
 Turku, Finland.
 jyrki.piilo@utu.fi*

SABRINA MANISCALCO

*Turku Centre for Quantum Physics, Department of Physics and Astronomy, University of
 Turku, FI-20014 Turun yliopisto,
 Turku, Finland.
 smanis@utu.fi*

Received Day Month Year
 Revised Day Month Year

We investigate the dynamics of quantum and classical correlations in a system of two qubits under local colored-noise dephasing channels. The time evolution of a single qubit interacting with its own environment is described by a memory kernel non-Markovian master equation. The memory effects of the non-Markovian reservoirs introduce new features in the dynamics of quantum and classical correlations compared to the white noise Markovian case. Depending on the geometry of the initial state the system can exhibit frozen discord and multiple sudden transitions between classical and quantum decoherence [L. Mazzola, J. Piilo, S. Maniscalco, Phys. Rev. Lett. **104**, 200401 (2010)]. We provide a geometric interpretation of those phenomena in terms of the distance of the state under investigation to its closest classical state in the Hilbert space of the system.

Keywords: discord; classical correlation; non-Markovian.

1. Introduction

In the last two decades lots of interest have been devoted to the definition and understanding of correlations in quantum systems. From the first formalization

*laumaz@utu.fi

of the separability problem by Werner¹ a number of fundamental results have been found in this field. In particular recently a big deal of attention has been devoted to the definition and study of quantum and classical correlations in quantum systems. The works of Ollivier and Zurek², and Henderson and Vedral³ had marked the beginning of a new line of research shifting the attention from the entanglement vs. separability dichotomy to the quantum vs. classical paradigm. The fact that separable states can be prepared via local operations and classical communications between two parties has often been interpreted as a signature of classicality. However, there are quantum correlations which are not captured by entanglement. For example the state $\rho = 1/2(|0\rangle\langle 0|_A \otimes |-\rangle\langle -|_B + |+\rangle\langle +|_A \otimes |1\rangle\langle 1|_B)$ with $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$ is separable, nevertheless it cannot be described by classical means, i.e., by a classical bivariate probability distribution. The reason for that is hidden in the non-orthogonality of the states of A and B , and consequently in the impossibility to locally distinguish the states of each subsystem.

Many different measures have been proposed to describe such more-general-than-entanglement quantum correlations^{3,4,5,6,7}, among them probably the most popular one is the quantum discord². The discord, defined as the difference between the quantum generalization of two equivalent formulations of classical mutual information, involves an optimization procedure over the set of measurements on a given subsystem. Therefore mathematical investigations have been carried to find analytical expressions for the discord in different types of systems, as in qubits^{8,9} and harmonic oscillators^{10,11}. The set of zero discord states has been shown to be measure zero and nowhere dense¹², moreover a new class of multipartite separable states, the pseudo-classical states, has been introduced and showed to have interesting properties¹³. Proposals to witness the presence of quantum discord have appeared in Ref. ^{14,15}. The role of quantum discord has been investigated in quantum information tasks as the power-of-one-qubit protocol¹⁶ or in the Grover search algorithm¹⁷, posing the possibility of using quantum discord as a new, more general than entanglement, quantum computation resource. The dynamics of quantum discord has been studied even in quantum biology, specifically in light-harvesting complex¹⁸.

A very active area of investigation of the discord is its behaviour under decoherence, i.e. the dynamics of quantum and classical correlations in open quantum systems^{19,20,21,22,23,24,25,26,27,28,29,30}. In particular in Ref. ³¹ we have discovered that quantum correlations, quantified by quantum discord, can be completely unaffected by decoherence for long intervals of time. Specifically, we have found that an open system of two qubits, in Markovian dephasing channels can undergo, what we have called a sudden transition between classical and quantum decoherence, namely a transition between a “classical decoherence” phase in which quantum correlation is frozen, while classical correlation is lost, and a “quantum decoherence” regime in which quantum correlation is deteriorated while classical correlation remains constant.

Here we study the time evolution of classical and quantum correlations for a system of two identical qubits under local non-dissipative non-Markovian channels. We use a well established model describing the dynamics of a two-level system in a dephasing channel in which white noise is replaced by colored noise³². There, the interaction with the environment is schematized by random telegraph signal noise. We study the non-trivial effects the non-Markovianity of the system brings to light. As in the Markovian case, there exists a class of initial states for which the system exhibits the sudden transition between classical and quantum decoherence and the frozen discord^{31,30}. However, here multiple transitions can arise due to the memory effects of the environment.

The transition between the two dynamical regimes can be explained, in the light of the results of Modi *et al.*⁷. There, the discord is defined as the relative entropy with respect to the closest classical state, therefore frozen discord corresponds to a dynamical evolution in which the state under investigation maintains a fixed distance (relative entropy or trace distance) to its closest classical state. In this framework the sudden transition corresponds to a sudden change in the location of the closest classical state in the Hilbert space of the system.

The outline of the paper is the following. In Sec. II we introduce the physical system under investigation³². In Sec. III we recall the concepts of quantum discord and classical correlation and study the dynamics of those quantities in our specific system. In Sec. IV we illustrate the interpretation of the phenomenon of frozen discord in terms of the distance to the closest classical state set. Section V summarizes and concludes the paper.

2. The model

We begin by introducing the physical model under study: two non-interacting qubits under local identical colored noise dephasing channels. Since the two qubits are locally interacting with their own environments, and are not coupled between each other, they have independent dynamical evolutions. Therefore we can first consider the dynamics of the single qubit, and then derive the dynamics of the composite two-qubit system.

A generic memory kernel master equation can be written in the form

$$\dot{\rho} = K\mathcal{L}\rho, \quad (1)$$

where ρ is the density operator of the small system of interest, in our case a qubit, \mathcal{L} is a Lindblad superoperator describing the dynamics induced by the interaction with the environment, K is an integral operator acting in the following way $K\phi = \int_0^t k(t-t')\phi(t')dt'$, and $k(t-t')$ is the kernel function determining the type of memory of the reservoir.

A master equation of this form arises when considering a two-level system subjected to random telegraphic noise. A model like this describes, for example, a spin in presence of a magnetic field, having constant intensity but inverting its

4 *Laura Mazzola, Jyrki Piilo and Sabrina Maniscalco*

sign randomly in time. It is possible to write a time-dependent phenomenological Hamiltonian for this kind of system:

$$H(t) = \hbar \sum_{i=1}^3 \Gamma_i(t) \sigma_i \quad (2)$$

here σ_i are the Pauli operators and $\Gamma_i(t)$ are independent random variables. We consider the general case of noise in the three directions. Each random variable can be written as $\Gamma_i(t) = a_i n_i(t)$. The random variable $n_i(t)$ has a Poisson distribution with a mean equal to $t/2\tau_i$, while a_i is a coin-flip random variable assuming the values $\pm a_i$. By considering the von Neumann equation $\dot{\rho} = -(\imath/\hbar)[H, \rho]$, one can find a formal solution for the qubit density matrix operator of the form

$$\rho(t) = \rho(0) - \imath \int_0^t \sum_k \Gamma_k(s) [\sigma_k, \rho(s)] ds. \quad (3)$$

When inserting such a formal solution back into the von Neumann equation and performing a stochastic average, one obtains the following memory kernel master equation

$$\dot{\rho}(t) = - \int_0^t \sum_k \exp(-(t-t')/\tau_k) a_k^2 [\sigma_k, [\sigma_k, \rho(t')]] dt' \quad (4)$$

in which the memory kernel comes from the correlation functions of the random telegraph signals $\langle \Gamma_j(t) \Gamma_k(t') \rangle = a_k^2 \exp(-|t-t'|/\tau_k) \delta_{jk}$.

The dynamics of this system and in particular the conditions of complete positivity of the map corresponding to such a master equation have been studied in detail by Daffer *et al.* in Ref. ³². In particular they demonstrated that complete positivity is assured when two of the a_i are zero, representing the physical situation of noise in only one direction. This is the case we are focusing here. In that work they also provided a Kraus operator representation of the map $\Phi_t(\rho) = \sum_k A_k^\dagger \rho A_k$. Depending on which of the $a_i = a$ is non zero the map has Kraus operators

$$\begin{aligned} A_i &= \sqrt{[1 - \Lambda(\nu)]/2} \sigma_i, \\ A_j &= 0, \quad A_k = 0, \\ A_4 &= \sqrt{[1 + \Lambda(\nu)]/2} I, \end{aligned} \quad (5)$$

where $\Lambda(\nu) = \exp \nu [\cos(\mu\nu) + \sin(\mu\nu)/\mu]$, $\mu = \sqrt{(4a\tau)^2 - 1}$, $\nu = t/2\tau$ is a dimensionless time, $i = 1, 2, 3$ is the direction of noise, and j and k the directions in which there is no noise. So by changing the direction of the noise we get a colored noise bit flip, bit-phase flip or phase flip channel, respectively. Having the Kraus operators of a single qubit, one can write the evolution of the generic state of the two qubits each interacting locally with its own environment as

$$\Phi_t(\rho_{AB}) = \sum_{i,j} A_i^{(A)} A_j^{(B)} \rho_{AB} A_i^{(A)\dagger} A_j^{(B)\dagger}, \quad (6)$$

where $\Phi_t(\cdot)$ is the completely positive trace preserving map ruling the evolution of the system.

It is worth mentioning that, even though in general the mathematical structure of the memory kernel master equations in Eq. (4) do not guarantee the presence of non-Markovian features in the dynamics³³, however the quantum process described by $\Phi_t(\cdot)$ is non-Markovian according to the measure of non-Markovianity of Ref.³⁴.

3. Dynamics of quantum and classical correlations

The physical quantities we want to investigate here are the quantum and classical correlations between the two qubits. We use the quantum discord as a measure of quantum correlations.

The idea behind quantum discord is to exploit the difference between quantum extensions of classically equivalent concepts to evaluate the “quantumness” of a quantum system. In particular in classical information theory there are two equivalent expressions for the mutual information of a bipartite system. However, when pursuing the quantum analogue, these two formulations differ, and one can use the mismatch between the two quantum extensions of classical mutual information to assess quantum correlations.

The first quantum generalization of mutual information is the so called quantum mutual information:

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \quad (7)$$

where ρ_{AB} is the density matrix of the total system, $\rho_{A(B)}$ is the reduced density matrix of subsystem $A(B)$, and $S(\rho) = -\text{Tr}\{\rho \log_2 \rho\}$ is the von Neumann entropy. This is generally accepted as the measure for the total amount of correlations (quantum and classical) of a quantum system^{5,35}.

The second extension of mutual information requires the generalization of the concept of conditional entropy. Indeed, performing measurements on system B affects our knowledge of system A, in particular how much system A is modified by the measurement depends on the choice of the measurement performed on B. Here the measurement B_k is considered of von Neumann type and it is described by a complete set of orthonormal projectors Π_k corresponding to outcome k . So the conditional density operator, which is the quantum state of the total system conditioned on the measurement outcome labeled by k , becomes $\rho_k = (I \otimes B_k)\rho_{AB}(I \otimes B_k)/p_k$ where $p_k = \text{Tr}\{(I \otimes B_k)\rho_{AB}(I \otimes B_k)\}$, and I is the identity operator for subsystem A. Defining the quantum analog of conditional entropy as $S(\rho_{AB}|\{B_k\}) = \sum_k p_k S(\rho_k)$ we can introduce the second quantum extension of mutual information.

$$J(\rho_{AB}|B_k) = S(\rho_A) - S(\rho_{AB}|B_k). \quad (8)$$

In Ref.³ Henderson and Vedral have shown that the maximum of this quantity over all the possible set of measurements can be interpreted as a measure of classical

6 *Laura Mazzola, Jyrki Piilo and Sabrina Maniscalco*

correlations of the state

$$C(\rho_{AB}) \equiv \max_{\{\Pi_j\}} [J(\rho_{AB}|B_k)]. \quad (9)$$

Therefore, the difference between the the quantum mutual information $I(\rho_{AB})$, describing total correlations, and the measurement-based definition of quantum mutual information $C(\rho_{AB})$, measuring classical correlations, defines the so-called quantum discord

$$D(\rho_{AB}) = I(\rho_{AB}) - C(\rho_{AB}). \quad (10)$$

In the following we investigate the dynamics of these quantities in our non-Markovian dynamical model for a particular class of states. The states we consider have maximally mixed marginals and have the form

$$\rho_{AB} = \frac{1}{4} \left(1_{AB} + \sum_{i=1}^3 c_i \sigma_i^A \sigma_i^B \right), \quad (11)$$

where $\sigma_i^{A(B)}$ are the Pauli operators in direction i acting on A (B), c_i are real numbers such that $0 \leq |c_i| \leq 1$ for every i , and 1_{AB} the identity operator of system AB. This class of states can equivalently be written in the form of Bell diagonal states

$$\begin{aligned} \rho_{AB}(t) = & \lambda_{\Psi}^{+}(t) |\Psi^{+}\rangle \langle \Psi^{+}| + \lambda_{\Phi}^{+}(t) |\Phi^{+}\rangle \langle \Phi^{+}| \\ & + \lambda_{\Phi}^{-}(t) |\Phi^{-}\rangle \langle \Phi^{-}| + \lambda_{\Psi}^{-}(t) |\Psi^{-}\rangle \langle \Psi^{-}|, \end{aligned} \quad (12)$$

where

$$\lambda_{\Psi}^{\pm}(t) = [1 \pm c_1(t) \mp c_2(t) + c_3(t)]/4, \quad (13)$$

$$\lambda_{\Phi}^{\pm}(t) = [1 \pm c_1(t) \pm c_2(t) - c_3(t)]/4, \quad (14)$$

and $|\Psi^{\pm}\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, $|\Phi^{\pm}\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ are the four Bell states.

The expressions of the classical and quantum correlations for this class of states are given by ⁸

$$C(\rho_{AB}) = \sum_{k=1}^2 \frac{1 + (-1)^k \chi}{2} \log_2(1 + (-1)^k \chi) \quad (15)$$

$$D(\rho_{AB}) = 2 + \sum_{k,l} \lambda_k^l(t) \log_2 \lambda_k^l(t) - C(\rho_{AB}), \quad (16)$$

where $\chi(t) = \max\{|c_1(t)|, |c_2(t)|, |c_3(t)|\}$, $k = \Psi, \Phi$, and $l = \pm$.

Since the dynamical evolution map $\Phi_t(\cdot)$ of Eq. (6) does not change the form of the state in Eq. (11), classical and quantum correlations are given by those expressions throughout all the time evolution.

As already noticed at the end of Sec. II, the interaction with the environment gives rise to a bit flip, bit-phase flip or phase flip channel when the direction of the noise is along x, y or z (in Eq. (5) $i = 1, 2, 3$), respectively. Using the Kraus

operators in Eq. (5) and the dynamical evolution of the state in Eq. (6) we find that the parameters $c_i(t)$ evolve in the following way

$$c_i(\nu) = c_i(0), \quad c_j(\nu) = c_j(0)\Lambda(\nu)^2, \quad c_k(\nu) = c_k(0)\Lambda(\nu)^2, \quad (17)$$

where $\nu = t/2\tau$ is dimensionless time, i is the direction of the noise, and j and k the other two directions. We focus on the phase flip channel (which means $i = 3$) case since completely equivalent results are found for the other two types of channels. The corresponding results for bit flip (bit-phase flip) channel are simply found exchanging c_3 with c_1 (c_2).

As in the Markovian case^{23,31} one can distinguish three different dynamical regimes, depending on the relations between c_1 , c_2 , and c_3 .

First regime: $|c_3(0)| \geq |c_1(0)|, |c_2(0)|$. In this regime, classical correlations remain constant in time. This is clearly displayed by Eq. (15), where $\chi(t)$ is equal to $|c_3(0)|$ which is constant during all the time evolution. The quantum mutual information characterizing the total correlations displays damped oscillations, therefore quantum discord is also exhibiting damped oscillations, tending asymptotically to zero. No discontinuities appear in quantum or classical correlations dynamics.

Second regime: $c_3 = 0$. Here, all the three types of correlations (quantum, classical, total) display damped oscillations and tend asymptotically to zero, meaning that the state of the system tends asymptotically to a product state.

Third regime: $|c_3(0)| < |c_1(0)|$ and/or $|c_2(0)|$. This is the region displaying most interesting dynamical features. Initially, classical correlations decay till to a certain point in time, and then become abruptly constant²⁷, and at that very same time the quantum discord changes its decay rate. In Ref.³¹ we demonstrated that the initial decay rate can be even equal to zero. This is mathematically explained observing the analytic formula of classical correlation in Eq. (15): initially $\chi(t)$ is equal to $\max\{|c_1(t)|, |c_2(t)|\}$ with $c_1(t)$ and $c_2(t)$ oscillating functions, then at the sudden change point $|c_3(0)|$ becomes bigger than $\max\{|c_1(t)|, |c_2(t)|\}$, therefore $\chi(t)$ is now equal to $|c_3(0)|$ which is constant in time, hence classical correlation becomes constant.

As time passes new features may appear in the dynamics of correlations. In fact if during the time evolution $|c_1(t)|$ or $|c_2(t)|$ turns bigger than $|c_3(0)|$ classical correlation is not constant anymore, and starts again oscillating until it becomes again abruptly constant. Clearly, sudden changes in the dynamics of discord take place in correspondence of the discontinuities of $\chi(t)$. Within this regime we can distinguish two different behaviours for the dynamics of the quantum correlation: the frozen behaviour, and the sudden change dynamics.

(i) Frozen discord with sudden changes displayed in Fig. 1 (a). The dynamics displays multiple sudden transitions: classical correlations at first decrease, while the discord remains constant unaffected by decoherence, then at a sudden transition point classical correlation becomes constant and discord oscillates, and then again discord becomes constant while classical correlation oscillates. The initial state in Fig. 1 (a) is of the form $\rho_{AB} = (1 + c_3)/2|\Psi^+\rangle\langle\Psi^+| + (1 - c_3)/2|\Phi^+\rangle\langle\Phi^+|$ with $c_3 =$

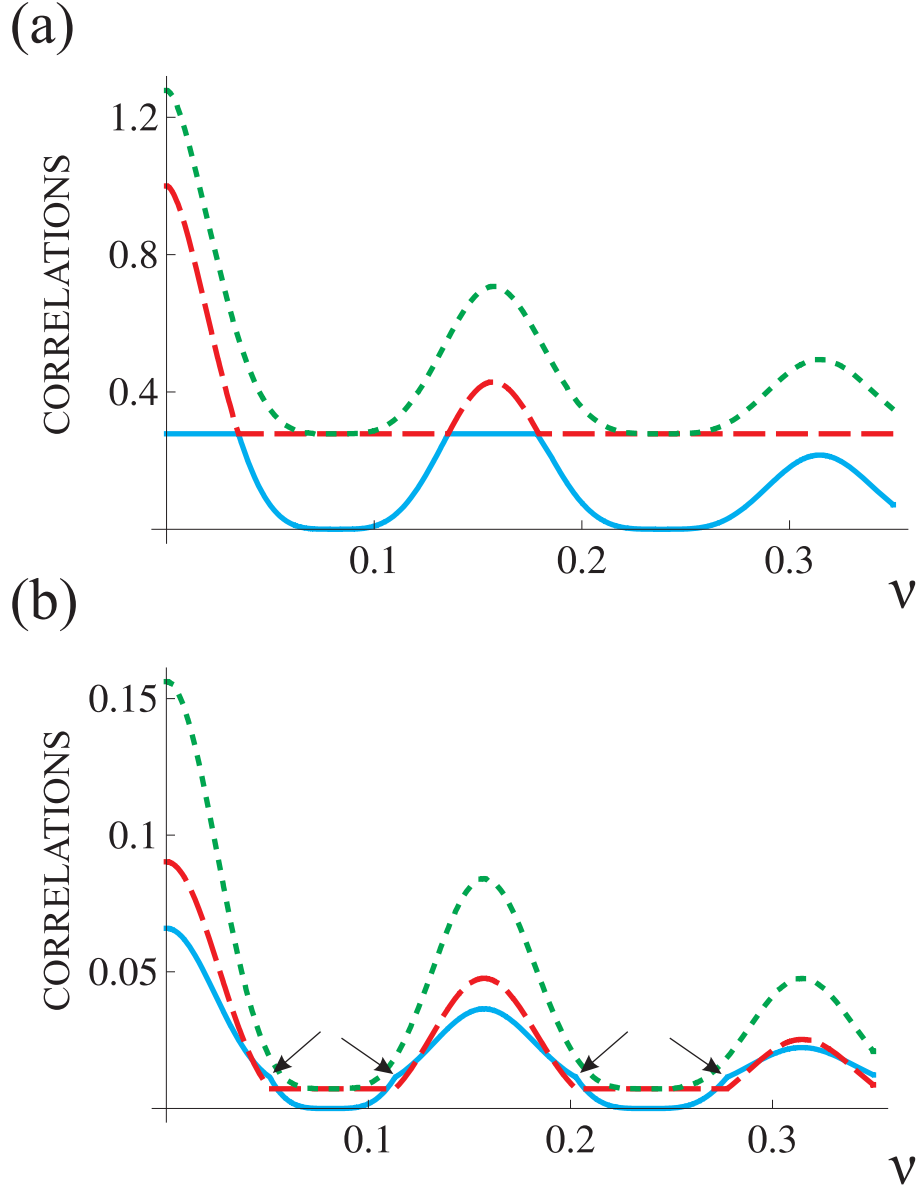


Fig. 1. (Color online) Dynamics of mutual information (green dotted line), classical correlations (red dashed line) and quantum discord (blue solid line) of two qubits locally interacting with non-Markovian dephasing channels as a function of scaled time $\nu = t/2\tau$ with $\tau = 5$ s and $a = 1$ s $^{-1}$. (a) Frozen discord and multiple transition regime, here we have set $c_1(0) = 1$, $c_2(0) = -0.6$ and $c_3 = 0.6$. (b) Sudden change regime, obtained setting $c_1(0) = 0.35$, $c_2(0) = -0.3$ and $c_3 = 0.1$; the arrows emphasize the sudden change points.

0.6, however a more general class of states exhibits frozen discord and (multiple) sudden transitions. This class of states is of the form of Eq. (11) where $c_{1(2)}(0) = k$, $c_{2(1)}(0) = -c_3 k$ and $c_3(0) = c_3$ with k real and $|k| > |c_3|$. For this class of states the quantum mutual information describing the total correlation of the system can be cast in a very simple form:

$$\begin{aligned} \mathcal{I}[\rho_{AB}(t)] = & \sum_{j=1}^2 \frac{1 + (-1)^j c_3}{2} \log_2[1 + (-1)^j c_3] \\ & + \sum_{j=1}^2 \frac{1 + (-1)^j c_{1(2)}(t)}{2} \log_2[1 + (-1)^j c_{1(2)}(t)], \end{aligned} \quad (18)$$

where the first constant term represents either the frozen discord or the constant classical correlation depending on the dynamical phase of the system.

(ii) The sudden change regime without frozen discord displayed in Fig. 1 (b). At first both classical and quantum correlations decrease till, at a sudden change point (emphasized in the figure by the first arrow on the left), classical correlation becomes constant while quantum correlation exhibits a discontinuous change in the amplitude of the damped oscillations. At the following sudden change point, indicated by the second arrow, classical correlation starts again to oscillate and quantum discord changes back to its previous rate of oscillation.

We mention in passing that, for all the regions of parameters, entanglement exhibits damped oscillations, similar to those of the quantum mutual information, eventually showing sudden death ³⁶.

4. Geometrical interpretation of frozen discord

In Ref. ⁷ Modi *et al.* propose a new definition for the correlations of a quantum system. Their idea is to put all the types of correlations on the same footing defining correlations in terms of distances (measured by relative entropy) between states. In analogy to the relative entropy of entanglement, quantum discord is defined as the distance of the state under study to its closest classical state, and classical correlation is the distance from such a classical state to its closest product state.

In general the definition by Modi *et al.* does not coincide with the original operational one, however, for Bell-diagonal state the two formulations lead to the same results. Therefore we can interpret our results in the light of the correlations as distance measure approach.

As already noted in Ref. ³¹, given a Bell-diagonal state $\rho = \sum_{i=1}^4 \lambda_i |\Phi_i\rangle\langle\Phi_i|$ with $|\Phi_i\rangle$ the Bell states and λ_i , (taken in decreasing order) the weights of each component, the sudden transition between classical and quantum decoherence takes place when the weight λ_3 of component Φ_3 becomes larger than the weight λ_2 of component Φ_2 . This can be understood noting that the expression for the closest

10 *Laura Mazzola, Jyrki Piilo and Sabrina Maniscalco*

classical state to Bell-diagonal states is

$$\rho_{\text{cl}}(t) = \frac{q(t)}{2} \sum_{i=1,2} |\Psi_i\rangle\langle\Psi_i| + \frac{1-q(t)}{2} \sum_{i=3,4} |\Psi_i\rangle\langle\Psi_i|, \quad (19)$$

where $q(t) = \lambda_1(t) + \lambda_2(t)$. Therefore when λ_3 becomes bigger than λ_2 the expression of the closest classical state changes abruptly. It is actually of interest to ask where those classical states are located in the Hilbert space of the system, and whether such a class of classical states display a particular geometry.

To fix the ideas we first focus on the Markovian case illustrated in Fig. 2 (a). There the solid black line represents the trajectory of the state $\rho(t)$ in a schematized Hilbert space, and the dotted red line represents the trajectory drawn by the closest classical state $\chi_\rho^{CD}(t)$ (where CD stands for constant discord). In terms of the distance (relative entropy or trace distance), this trajectory is parallel to the one traced by $\rho(t)$ meaning that the discord between the two lines is constant. The green square on the $\rho(t)$ trajectory indicates the state ρ_{ST} at which the sudden transition takes place, in correspondence of $\lambda_2 = \lambda_3$. The state ρ_{ST} has two different classical states with equal distance: one is the last state at the end of the red dotted line, i. e., $\chi_\rho^{CD}(t)$, the other one is indicated by the red sphere at the right end of the black line $\chi_\rho^{DD}(t)$. Since $\rho(t)$ keeps traveling the black line from left to right, after the transition $\chi_\rho^{DD}(t)$ (DD meaning decaying discord) becomes the closest classical state. Interestingly $\chi_\rho^{DD}(t)$ is also the asymptotic state of the dynamics and does not evolve in time.

The non-Markovian case is displayed in Fig. 2 (b), there the meaning of lines and symbols, and the structure of the set of closest classical states remain the same as in Fig. 2 (a) but the time evolution of $\rho(t)$ changes. Due to the non-Markovian memory effects, the state of the system oscillates around the transition point and passes many times from both of the directions the transition state ρ_{ST} . After the first passage of $\rho(t)$ through ρ_{ST} from left to right, $\rho(t)$ hits in a finite time $\chi_\rho^{DD}(t)$, which in the Markovian case was reached only in the asymptotic limit. Now the memory of the reservoir comes into play and the state travels back part of the trajectory previously drawn, this is pictured by the thick dashed blue line partially overlapping with the black one. This time the state ρ_{ST} is passed from right to left, so $\rho(t)$ enters again in the constant discord region. Eventually the direction of the trajectory of $\rho(t)$ in the Hilbert space changes again and the state enters again the changing-discord-region and remaining there.

5. Conclusive Remarks

In this paper we have studied the dynamics of quantum and classical correlations for a system of two qubits locally interacting with non-Markovian non-dissipative environments. To solve the dynamics of the single qubits we extended the model by Daffer *et al.* to the two-qubit case, when each qubit is subjected to a local colored noise dephasing channel. For initial Bell-diagonal states the dynamics of correlations exhibits, three different dynamical regimes: (i) constant classical correlations;

(ii) damped oscillations of quantum and classical correlations (iii) sudden change behavior with possibility for frozen discord including multiple sudden transitions of decoherence. The memory effects of the non-Markovian environment bring into play non-trivial dynamical features and allow multiple transition points between the classical and quantum decoherence.

We have also proposed a geometrical interpretation of the phenomenon of frozen discord in terms of trajectory of the state under investigation and its closest classical state. In the transition point between the classical and quantum decoherence, the location of the closest classical state changes in discontinuous way. In the Markovian case, this transition point is crossed only once, where as in the non-Markovian case, the memory effects allow several crossings to both directions.

Very recently Jin-Shi et al.³⁷ have observed the dynamics of the correlations of two photons in a non-Markovian dephasing environment and their results display the same features as described here. In general, we think that results for dephasing reservoirs shown here and the corresponding experimental results³⁷ present a considerable step towards more comprehensive understanding of the dynamics of quantum correlations in the open system context.

Acknowledgments

This work has been supported by the Academy of Finland (Project No. 133682), the Emil Aaltonen Foundation, the Finnish Cultural Foundation, and the Magnus Ehrnrooth Foundation. S.M. also thanks the Turku Collegium of Science and Medicine for financial support.

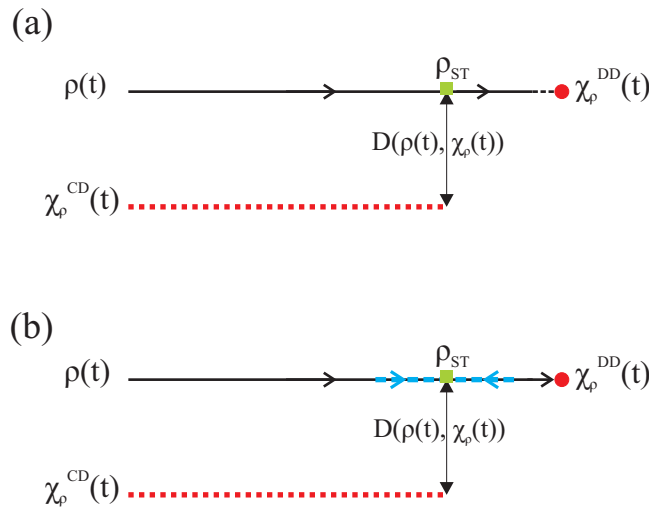


Fig. 2. (Color online) Schematization of the trajectories of both the state of the system and its closest classical state in the Hilbert space in the (a) Markovian, (b) non-Markovian case.

12 *Laura Mazzola, Jyrki Piilo and Sabrina Maniscalco*

References

1. R. F. Werner, *Phys. Rev. A* **40**, 4277 (1989).
2. H. Ollivier and W. H. Zurek, *Phys. Rev. Lett.* **88**, 017901 (2001).
3. L. Henderson and V. Vedral, *J. Phys. A* **34**, 6899 (2001).
4. J. Oppenheim, M. Horodecki, P. Horodecki, and R. Horodecki, *Phys. Rev. Lett.* **89**, 180402 (2002).
5. B. Groisman, S. Popescu, and A. Winter, *Phys. Rev. A* **72**, 032317 (2005).
6. S. Luo, *Phys. Rev. A* **77**, 022301 (2008).
7. K. Modi, T. Paterek, W. Son, V. Vedral, and M. Williamson, *Phys. Rev. Lett.* **104**, 080501 (2010).
8. S. Luo, *Phys. Rev. A* **77**, 042303 (2008).
9. M. Ali, A.R.P. Rau, and G. Alber, *Phys. Rev. A* **81**, 042105 (2010).
10. P. Giorda and M. Paris, *Phys. Rev. Lett.* **105**, 020503 (2010).
11. G. Adesso and A. Datta, *Phys. Rev. Lett.* **105**, 030501 (2010).
12. A. Ferraro, L. Aolita, D. Cavalcanti, F.M. Cucchietti, and A. Acín, *Phys. Rev. A* **81**, 052318 (2010).
13. L. Chen, E. Chitambar, K. Modi, and G. Vacanti, arXiv:1005.4348.
14. B. Dakić, V. Vedral, Č. Brukner, *Phys. Rev. Lett.* **105**, 190502 (2010).
15. B. Bylicka, D. Chruściński, *Phys. Rev. A* **81**, 062102 (2010).
16. A. Datta, A. Shaji, and C. Caves, *Phys. Rev. Lett.* **100**, 050502 (2008).
17. J. Cui and H. Fan, *J. Phys. A: Math. Theor.* **43**, 045305 (2010).
18. K. Bradler, M. M. Wilde, S. Vinjanampathy, D. B. Uskov, arXiv:0912.5112.
19. W. H. Zurek, *Phys. Rev. A* **67**, 012320 (2003).
20. M. Horodecki *et al.*, *Phys. Rev. A* **71**, 062307 (2005).
21. C. A. Rodriguez-Rosario *et al.*, *J. Phys. A* **41**, 205301 (2008).
22. M. Piani, P. Horodecki, and R. Horodecki, *Phys. Rev. Lett.* **100**, 090502 (2008).
23. J. Maziero *et al.*, *Phys. Rev. A* **80**, 044102 (2009).
24. A. Datta and S. Gharibian, *Phys. Rev. A* **79**, 042325 (2009).
25. M. Piani *et al.*, *Phys. Rev. Lett.* **102**, 250503 (2009).
26. T. Werlang *et al.*, *Phys. Rev. A* **80**, 024103 (2009).
27. J. Maziero *et al.*, *Phys. Rev. A* **81**, 022116 (2010).
28. F. F. Fanchini *et al.*, *Phys. Rev. A* **81**, 052107 (2010).
29. R. Vasile, P. Giorda, S. Olivares, M. G. A. Paris, and S. Maniscalco, *Phys. Rev. A* **82**, 012313 (2010).
30. Jin-Shi Xu *et al.*, *Nat. Commun.* **1**, 7 (2010).
31. L. Mazzola, J. Piilo, and S. Maniscalco, *Phys. Rev. Lett.* **104**, 200401 (2010).
32. S. Daffer, K. Wódkiewicz, J. D. Cresser, and J. K. McIver, *Phys. Rev. A* **70**, 010304(R) (2004).
33. L. Mazzola, E.-M. Laine, H.-P. Breuer, S. Maniscalco, and J. Piilo, *Phys. Rev. A* **81**, 062120 (2010).
34. H.-P. Breuer, E.-M. Laine, and J. Piilo, *Phys. Rev. Lett.* **103**, 210401 (2009).
35. B. Schumacher and M. D. Westmoreland, *Phys. Rev. A* **74**, 042305 (2006).
36. T. Yu and J. H. Eberly, *Phys. Rev. Lett.* **93**, 140404 (2004).
37. Jin-Shi Xu, Chuan-Feng Li, Cheng-Jie Zhang, Xiao-Ye Xu, Yong-Sheng Zhang, and Guang-Can Guo, arXiv:1005.4510.